Solution of a Large Scale Linear Least Squares System Through QR Updating Method

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Abstract

Large scale systems of linear equations are often difficult to solve because of their shear size. This is often coupled with ill-conditioning, loss of rank and so on. By concentrating on particular parts of the problem matrix, the problem may be made amenable to solution. Here, we consider solving linear systems in the sense of least squares problem, i.e. $min_x||Ax - b||_2$, where $A \in \mathcal{R}^{m\times n}$ and $b \in \mathcal{R}^m$. The suggested approach relies on repeated updating of the QR factor of a sub-matrix of A. The original matrix is first reduced by removing some columns. It is then reduced by removing some rows from it. The right-hand side is reduced accordingly. When a manageable system is obtained, it is solved by a QR method. Columns are then appended and the QR factor is updated. This intermediary system is solved. Rows are then appended and the intermediary solution is updated to get the solution to the overall system. This approach can be applied in a single pass or a number of passes as the situation may require. It can also be applied to not necessarily large scale systems particularly when a QR factor of a sub-matrix of the original problem matrix is already known. Note that, independently of number of passes used, when the last rows are appended, the algorithm of George & and Heath' is used. In this talk I will report numerical results obtained on randomly generated problems solved with a Matlab implementation of the of the suggested algorithm.