Perfect packings in graphs

Andrew Treglown

Given two graphs H and G, a perfect H-packing in G is a collection of vertex disjoint copies of H in G that cover all the vertices of G. Perfect H-packings are generalisations of perfect matchings (which correspond to the case when H is a single edge).

In the case when H is an edge, Tutte's theorem characterises those graphs which have a perfect H-packing. However, for other connected graphs H no characterisation is known. It is natural therefore to ask for simple sufficient conditions which ensure the existence of a perfect H-packing. One such result is a theorem of Hajnal and Szemerédi which states that a graph G on n vertices, where r divides n, has a perfect K_r-packing provided that the minimum degree of G is at least $(1-1/r)n$. It is easy to see that the minimum degree condition here is best possible. Kühn and Osthus characterised, up to a constant, the minimum degree condition that ensures a graph G contains a perfect H-packing for *any* graph H.

Ore-type degree conditions consider the sum of the degrees of non-adjacent vertices. Together with Kühn and Osthus we characterised asymptotically the Ore-type degree condition that ensures a graph G contains a perfect H-packing for any graph H. I will discuss this result and how, perhaps surprisingly, it takes a quite different form to the corresponding result concerning the minimum degree of a graph.