Surface Tension Driven Flow in a Half Plane

Alice Thompson

T We consider the case of a fat wedge of inviscid fluid , of angle $\pi - \epsilon$ where ϵ is small , in contact with a rigid wall along one edge . Initially the fluid is at rest . At some time $t = 0$, the contact angle at the tip of the wedge is suddenly changed to $\pi - \lambda\epsilon$. The resulting flow and motion of the contact point is determined by a balance of surface tension and inertia . As there are no geometric lengthscales imposed , we obtain a similarity solution with lengths scalings as $t^{2/3}$.

For arbitrary λ and ϵ , the resulting similarity equations are nonlinear on an unknown domain . In the limit $\epsilon \to 0$ with $\lambda = O(1)$, we can linearise the domain to a half plane , with the free surface displacement and velocity potential coupled to the flow via linear PDEs . We solve these equations numerically , with the aid of the boundary integral method , and also present asymptotic solutions for λ large and λ close to 1 .

For large λ , the solution breaks down into inner and outer regions , with the phase and amplitude of the capillary wave set in the inner region via the solution to the Wiener-Hopf dock problem . The decay of the mean free surface displacement matches into the outer region to determine the relationship between λ and the contact point position x_c . For λ close to 1 , the leading order problem can be solved exactly using Mellin transforms.