$\mathscr{C}(K)$ spaces with few operators

Iryna Schlackow

Let K be an infinite compact Hausdorff space and consider the Banach space $\mathscr{C}(K)$ of continuous real-valued functions on K. We are interested in the following question: how small can the space $\mathscr{L}(\mathscr{C}(K))$ of (bounded linear) operators on $\mathscr{C}(K)$ be? Clearly, for a continuous function g, the multiplication operator $gI\colon f \mapsto gf$ is a well-defined operator on $\mathscr{C}(K)$. In addition, it may be shown that there always exists an operator T that cannot be expressed in the form $T = gI + K$ with g continuous and K compact. On the other hand, in 2004 Piotr Koszmider constructed a space K such that every operator on $\mathscr{C}(K)$ has the form $T = gI + W$ with g a continuous function and W weakly compact. Spaces K with this property are now called Koszmider spaces.

In this talk we present some properties of spaces $\mathscr{C}(K)$ where K is a Koszmider space. Due to a simple algebraic structure of $\mathscr{L}(\mathscr{C}(K))$, our results follow directly from ring isomorphism theorems and are especially interesting when we restrict ourselves to spaces with no isolated points. We also describe topological properties of connected Koszmider spaces.

In addition to the above, we introduce a related notion of weakly Koszmider spaces and present some of their properties.