High Reynolds Number Flow in a Collapsible Channel

The motivation of this work is to understand self-excited oscillations that arise whenever a fluid is driven rapidly through an externally pressurized flexible tube. These instabilities are believed to have various physiological applications, such as wheezing in airways and Korotkov sound generation during sphygmomanometry. We consider two-dimensional high Reynolds number laminar flow of a Newtonian, incompressible fluid though a collapsible channel. A section of one wall of an otherwise rigid channel is replaced by a membrane with inertia M, under longitudinal tension T, with zero bending stiffness and subject to an external pressure P_{ext}. The segments upstream and downstream from the elastic wall are assumed to be infinitely long. The inviscid, large amplitude and long wavelength assumption allows the membrane motion to be coupled to the time-dependent behaviour of the core flow through a modified KdV equation [Pedley & Stephanoff, *J. Fluid Mech*, 160:337-367, 1985] of the form

$$\sigma A_{xxx} - 12\beta A_t - 36(AF)_x = 6\beta F_t + 36 F F_x.$$

$A(x,t)$ represents a lateral displacement of the core-flow streamlines in the y-direction and $F(x,t)$ defines the shape of the membrane according to the equation

$$P_{ext} - P = M F_{tt} - T F_{xx}, \quad if \quad 0 < x < 1,$$

$$F = 0, \quad \text{elsewhere.}$$

Therefore, the spatial dimension of the problem is reduced to one. The model is studied numerically. In certain parameter ranges the solutions reveal small-amplitude self-excited oscillations of the collapsible channel. These results are compared with numerical simulations using the two-dimensional Navier-Stokes equations at moderate Reynolds numbers.