Bracket quadratics as asymptotic bases for the integers

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One of the classical problems of additive number theory, known as Waring's problem, is to show that the k^{th} powers form a basis for the integers. That is, for any k there is some $s = s(k)$ such that every positive integer is a sum of s k^{th} powers. Lagrange's theorem, which says that every positive integer is a sum of four squares, is a special case of this. Waring's problem was first solved by Hilbert, and then a few years later Hardy and Littlewood supplied a new proof, using what is now known as their circle method.

I shall describe how to use a new variation of the circle method to show a Waring-type result: that the bracket quadratics $n\lfloor n\sqrt{2}\rfloor$ form an asymptotic basis for the integers. That is, there is some s so that every sufficiently large positive integer is a sum of s numbers of the form $n\lfloor n\sqrt{2}\rfloor$. The proof uses recent work of Green and Tao on the quantitative distribution of polynomial orbits on nilmanifolds. This is joint work with Ben Green.