STABILITY OF ANOSOV HAMILTONIAN STRUCTURES

WILL J. MERRY

Abstract. This talk will be about *twisted geodesic flows* on the tangent bundle TM of a closed Riemannian manifold M. Twisted geodesic flows should be thought of as being obtained from the standard geodesic flow by 'twisting' with a closed 2-form σ. Twisted geodesic flows are also known as *magnetic flows*; they are physically relevant as they can be viewed as modelling the motion of a particle under the effect of a magnetic field (which is represented by the closed form σ). The geodesic flow is a special case of a twisted geodesic flow; simply take $\sigma = 0$.

If $E : TM \to \mathbb{R}$ denotes the kinetic energy functional $E(v) = \frac{1}{2}|v|^2$, then the level sets $E^{-1}(k)$ are hypersurfaces in TM invariant under the twisted geodesic flow. It is an interesting and natural question (and one which I shall try to motivate during the talk) to ask under what conditions a hypersurface $E^{-1}(k)$ is *stable*. The aim of the talk is to sketch the proof of the following result:

Theorem. (Merry, Paternain '09)
Let (M, g) be a closed Riemannian manifold of dimension $n \geq 3$. Suppose g is negatively curved and strictly $1/4$-pinched. Then for any k sufficiently large the hypersurface $E^{-1}(k)$ is stable if and only if σ is exact.

This talk should be accessible to anyone who knows basic symplectic geometry; it is however especially aimed at people who attended Gabriel Paternain's 'The X-ray transform in Geometry and Dynamics' Part III course last year.