

Lagrangian knots and pseudoholomorphic curves

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Consider two Lagrangian embeddings of some surface into a symplectic 4-manifold. One can ask a number of different questions: Are they smoothly isotopic? Isotopic through Lagrangian embeddings? Is there even a Hamiltonian isotopy connecting them? This is known as the "Problem of Lagrangian knots in symplectic 4-manifolds", and it is not very well understood in general. Even in the case of linear symplectic 4-space (\mathbb{C}^2, ω_0) , only partial results are known. Roughly speaking, it is hard for a Lagrangian torus to be smoothly knotted, in the sense of not being smoothly isotopic to the Clifford torus (which may be regarded as the "unknot"). However, things are different in the symplectic category: There is at least one example of an exotic Hamiltonian isotopy class.

After giving a brief introduction to the problem, I will explain a possible new line of attack. The idea is to look at Lagrangian immersions into \mathbb{C}^2 as pseudoholomorphic curves and to use the language of moduli spaces.

The talk will not assume any particular knowledge about symplectic topology.